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SHORTER COMMUNICATIONS

A COMMENT ON THE TURBULENT FLOW VELOCITY PROFILE IN A CONCENTRIC ANNULUS

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NOMENCLATURE

- B, constant;
- k, mixing length constant;
- l, mixing length;
- p, $= (r_m/r_1)$ constant;
- r, radius;
- u, fluid velocity in direction of mean flow;
- u^*, $= \sqrt{(\tau_w/\rho)}$ friction velocity;
- u^+, $= u/u^*$ non-dimensional velocity;
- y, distance from a wall;
- y^+, $y(u^*/\nu)$ non-dimensional wall distance.

Greek symbols

- α, $= (r_2/r_1)$ radius ratio;
- ε, eddy diffusivity;
- ν, kinematic viscosity;
- ρ, mass density;
- τ, shear stress.

Subscripts

- w, wall;
- 1, inner region or inner wall;
- 2, outer region or outer wall;
- m, point of zero shear stress.

INTRODUCTION

A MOST interesting feature of the study of turbulent annular flow velocity profiles, is the complete failure of the conventional "Universal Velocity Profile" to correlate the results in the inner wall region. There is a wealth of reliable experimental data which lends support to the accuracy of the familiar "Law of the Wall" in the outer wall region, but to date there is no completely satisfactory counterpart for the inner wall region.

Recently Levy [1] derived expressions for the velocity profiles in turbulent concentric annular flow. In his analysis Levy assumed that Reichardt's [2] expression for eddy diffusivity in pipe flow was applicable in a modified form, to annular flow; the true parabolic shear stress distribution in annular flow was also used by Levy. Following an analysis similar to that used for the derivation of the turbulent flow laws for pipes and parallel plates. Levy obtained the following cumbersome expression for the non-dimensional velocity profile:

$$u^+ = \left\{ \frac{1}{k} \ln \left(1{\cdot}5 y^+ \cdot \frac{1+b}{1+2b^2} \right) + \left[\frac{2t(1-t)}{k(1+t)(2t-1} \right] \ln \left(\frac{1+b}{2} \right) + \tfrac{1}{2} \cdot \frac{t(1-t)(1-3t)}{k(1+t)[t^2 + \frac{1}{2}(1+t)^2]} \cdot \ln \left(\frac{1+2b^2}{3} \right) + \frac{6}{k(1+t)} \cdot \frac{\ln[b(1-t)+t]}{[(1-t)^2/36 - 1][(t/1-b)^2 + 2]} + \frac{\sqrt{2}}{k} \cdot \frac{(1-t)\,t}{(1+t)} \frac{\tan^{-1}(\sqrt{2}) - \tan^{-1} b.(\sqrt{2})}{t^2 + \frac{1}{2}(1-t)^2} + 14{\cdot}84 - \frac{1}{k} \ln 42 \right\} \quad (1)$$

where:

$$b = (r - r_m)/(r_w - r_m)$$
$$t = (r_m/r_w).$$

This expression is valid for both the inner and outer wall regions—the appropriate values being given to k.

Bearing in mind that for the prediction of heat-transfer results in annuli, we utilize the velocity profiles for both the inner and the outer regions, it is obvious that the expression derived by Levy would make the computation unnecessarily complex and involved.

This note presents a method of analysis which is basically the same as that of Levy's, but which leads to a very much simpler and more convenient result which is found to agree with experimental results as well as, if not better than, the cumbersome expression of Levy.

ANALYSIS

Levy's results can be greatly simplified if we assume, firstly that the shear stress is constant across the flow section and secondly, that the position of the radius of zero shear is given by the expression suggested by Leung *et al.* [3] for the radius of maximum velocity. As a result of some unpublished experiments by the author, it is confidently felt that the correlation by Leung *et al.* for the radius of maximum velocity represents the position of the radius of zero shear for fully turbulent flow even for large radius ratios. The more recent results of Brighton [4] and Lee [5], which encompass a wider range of radius ratios (up to Lee's $\alpha = 80{\cdot}7$) than originally given by Leung *et al.*, lend weight to Leung's suggested correlation.

Applying Reichardt's expression for the eddy diffusivity in pipe flow, to fully developed turbulent flow in a concentric annulus, we have for the outer region:

$$\frac{\varepsilon_2}{\nu} = \frac{k_2}{6}(r_2 - r_m)\frac{u_2^*}{\nu}\left[1 - \left(\frac{r - r_m}{r_2 - r_m}\right)^2\right] \times \left[1 + 2\left(\frac{r - r_m}{r_2 - r_m}\right)\right]. \quad (2)$$

With a corresponding expression for the inner region. Because the eddy diffusivity varies continuously across the section, then we can equate ε_1 and ε_2 at $r = r_m$.

Thus,

$$k_1 = \left\{k_2\frac{(r_2 - r_m)}{(r_m - r_1)}\sqrt{\frac{\tau_{w_2}}{\tau_{w_1}}}\right\}. \quad (3)$$

But from Leung *et al.* we have

$$r_m = r_1\left\{\frac{[1 + (\alpha)^{1-n}]}{[1 + (1/\alpha)^n]}\right\} = r_1 \times p. \quad (4)$$

$$n = 0{\cdot}343.$$

Carrying out a force balance on an annular element of fluid we find that

$$\left(\frac{\tau_{w_1}}{\tau_{w_2}}\right) = \frac{r_2}{r_1} \times \frac{(r_m^2 - r_1^2)}{(r_2^2 - r_m^2)}. \quad (5)$$

Substituting (4) and (5) into (3) gives

$$k_1 = k_2\left\{\frac{(\alpha - p)}{(p - 1)}\Big/\sqrt{\left(\frac{(\alpha^2 - p^2)}{\alpha(p^2 - 1)}\right)}\right\}. \quad (6)$$

Thus knowing α and k_2 we can find k_1. A value of 0·4 for k_2 correlates the outer region satisfactorily for all values of α—as would be expected because the structure of the flow in this region is very similar to that occurring in pipe flow.

Hence the "modified" Universal Velocity Profile for the inner region takes the form

$$u_1^+ = \frac{1}{k_1}.\ln y_1^+ + B_1.$$

B_1 is found by arranging for the curve to pass through $u^+ = 13, y^+ = 22$; this point is found to be the most suitable for the joining of the inner and outer regions based upon experimental results.

DISCUSSION

Figure 1 shows the variation of the modified mixing length

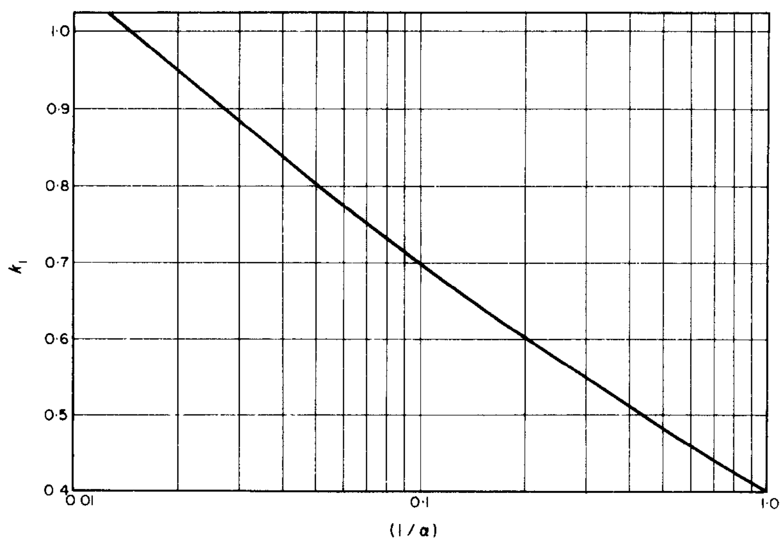

FIG. 1. Modified mixing length constant.

Figure shows the variation of the modified mixing length constant k_1, as calculated from equation (6) for a range of radius ratios from 1 to 100.

A summary of results in the form of a modified Universal Velocity Profile for the inner region is shown in Fig. 2 for values of α of 100, 30, 16, 8 and 1·5; together with the usual form of the velocity profile which is common to the outer region of all concentric annuli.

In Fig. 3 the predicted profiles are compared with the experimental results of Brighton [4] for a radius ratio of $\alpha = 16$, together with the curve predicted by Levy. There is seen to be a noticeable improvement with the present correlation.

From his work on pipe flow, Deissler [6] concluded that the effect of variable shear stress on velocity distribution was only very slight, and that the $u^+ - y^+$ relationship based upon constant shearing stress across the pipe, predicted the velocity profiles to within the experimental error.

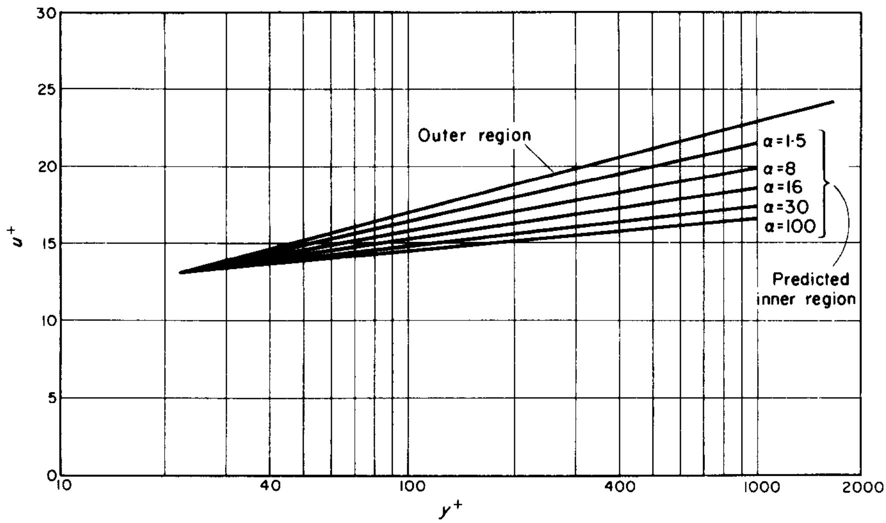

FIG. 2. Universal velocity profiles.

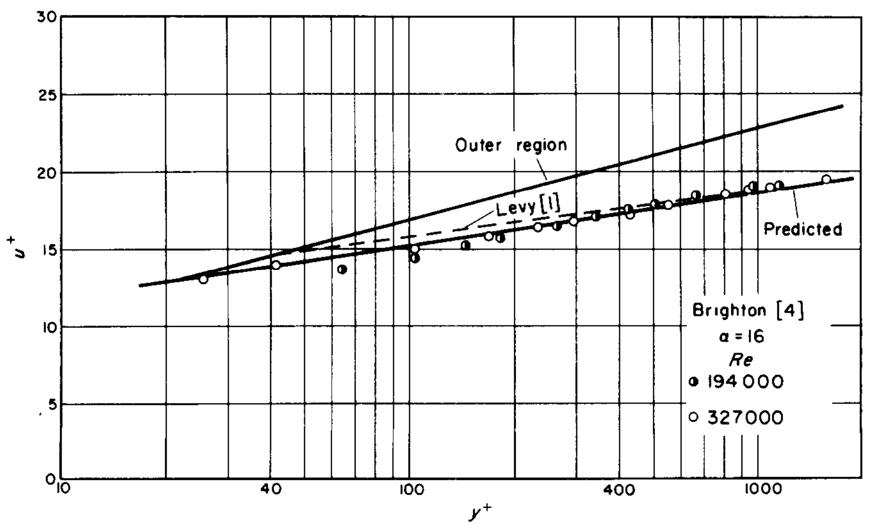

FIG. 3. Universal velocity profiles.

It would appear from the foregoing that the same is also true for annuli.

From this work it is apparent that the mixing length constant is not in fact a universal constant for all surfaces, as postulated by some workers, but is in fact a function of the radius or curvature ratio of the annulus to which it applies.

The method contained herein appears to give a simple, but reliable, prediction of the form of the $u^+ - y^+$ relationship for the inner region of annuli.

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ASYMMETRIC HEAT TRANSFER IN TURBULENT FLOW BETWEEN PARALLEL PLATES

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NOMENCLATURE

D_e, = 2s, equivalent diameter of the channel;
f, function;
h, convective heat-transfer coefficient;
Nu, Nusselt number (based on equivalent diameter);
Re, Reynolds number (based on equivalent diameter);
s, width of the channel;
T, temperature;
u, velocity;
q heat flux;
x, distance in the direction of flow;
y, distance from a wall

Greek symbols
α, dimensionless temperature as defined in equations (11) and (12);
γ, $= (q''_w/q'_w)$, ratio of the heat fluxes at the walls of the channel;
ϵ, total conductivity of heat.

Subscripts
PS, value in symmetrical heating case;
P, Q, refer to the walls of the channel;
w, wall value;
B, bulk value.

Superscripts
$'$, $''$, $'''$, refer to the thermal boundary conditions described in Fig. 1.

INTRODUCTION

A NUMBER of analytical studies [1–5] have been made of heat-transfer coefficients in flow between parallel plates with unequal heat fluxes at the two plates. An experiment has been reported by Barrow [4], but the results showed considerable scatter and its was difficult to confirm that the analysis given by Barrow in the same paper adequately described the variation in the heat-transfer coefficient (at one wall) with the ratio of the fluxes at the two walls, γ.

In most of the previous work, an analytical solution is first obtained for flow between the plates with heat transfer at one wall and the other wall insulated. Assumptions for the variations of the velocity and eddy diffusivity of heat across the channel are made. The case of asymmetric heat transfer

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